

Control Strategies for Space Boosters Using Air Collection Systems

H. G. Kauffman,* R. V. Grandhi,† W. L. Hankey,‡ and P. J. Belcher§
Wright State University, Dayton, Ohio 45435

A simple and efficient performance analysis method is developed for evaluating vehicle pitch and engine throttle controls to minimize booster fuel required to fill second-stage liquid oxygen (LOX) tanks and deliver the vehicle to the staging point. An optimization methodology finds a throttle schedule that controls both the air-breathing engine and the LOX collection rate. The altitude-velocity profile is derived from a variational calculus/energy management contouring method. Automatic adaptive-gain pitch-rate and throttle controls are developed. Results from a parametric study show that collecting on the run for an optimum schedule results in a 17% fuel savings over collection at a constant Mach number.

Nomenclature

A_0	= engine inlet streamtube area, ft ²
A_C	= engine inlet capture area, ft ²
C_D	= coefficient of drag
C_L	= coefficient of lift
C_R	= collection ratio of LOX to hydrogen fuel, \dot{W}_{LOX}/\dot{W}
C_ϕ	= engine throttle setting
D	= drag force, lb
E	= specific energy, $h + \frac{1}{2}V^2/g$, ft
F_{FCN}	= fuel integrand, \dot{W}/P_s , lb/ft
FA_{STOIC}	= stoichiometric H ₂ -fuel/air ratio
f_s	= fuel specific energy, P_s/\dot{W} , ft/lb
g	= gravity constant, ft/s ²
h	= height above sea level, ft
I_{SP}	= specific impulse, s
L_{FCN}	= LOX integrand, \dot{W}_{LOX}/P_s , lb/ft
L	= lift force, lb
M	= Mach number
m	= vehicle mass, slug
P_s	= specific excess power, $V(T-D)/\dot{W}$, ft/s
q	= dynamic pressure, lb/ft ²
R	= radius from Earth's center, ft
R_E	= Earth radius, ft
S	= vehicle wing surface reference area, ft ²
s	= distance downrange from vehicle launch point, n.mi.
T	= thrust, lb
t	= time, s
V	= flight velocity, ft/s
\dot{W}_A	= engine air flow, lb/s
\dot{W}	= engine fuel flow, lb/s
W	= vehicle weight, lb
α	= angle of attack, deg
α_0	= zero lift angle of attack, deg
Δ	= denotes an increment
γ	= flight-path angle, deg
ϕ	= fuel-to-air equivalence ratio

Introduction

THE last three decades have seen much attention in the design and analysis of air-breathing (AB) propulsion systems. Practical schemes to obtain high specific impulse up to orbital speed are being sought for Earth-to-orbit missions such that sizable payloads may be delivered to orbit. Some of the different schemes under study are 1) air-breathing propulsion to orbit, 2) combined air-breathing and rocket propulsion systems, and 3) air-breathing propulsion with ACES (Air Collection and Enrichment Systems), which provides oxidizer for rocket operation. Among these, ACES is of interest as a method of reducing the launch/payload mass ratio and increasing specific impulse for an Earth-to-orbit vehicle. This concept has the lowest takeoff weight (less than 1 million lb) of any other contender for the same payload delivery to orbit (150,000 lb), since it takes off with only liquid hydrogen (LH₂) fuel aboard and collects the required oxygen during flight.

The ACES vehicle takes off and accelerates on turbojet/ramjet engines. It carries air enrichment equipment that separates the oxygen from the nitrogen and liquefies and stores the oxygen in the tanks of the second stage. After collection, the airbreather uses a scramjet to accelerate to Mach 8 where the second stage separates for climb to orbit and the first stage returns to base. The second stage is rocket powered and uses the collected liquid oxygen (LOX) and stored liquid hydrogen fuel.

This paper addresses the performance optimization of a given ACES vehicle such that the hydrogen fuel is minimized to collect and fill the second-stage oxidizer tanks and deliver the second stage to the launch point. Some publications of interest to this problem are discussed next.

Nau¹ studied several two-stage reusable launch vehicle concepts for payload delivery to orbit. One of the vehicles was designed to take off with only LH₂ fuel aboard and to collect oxidizer (LOX) for use by the second stage climb to orbit. The second-stage rocket

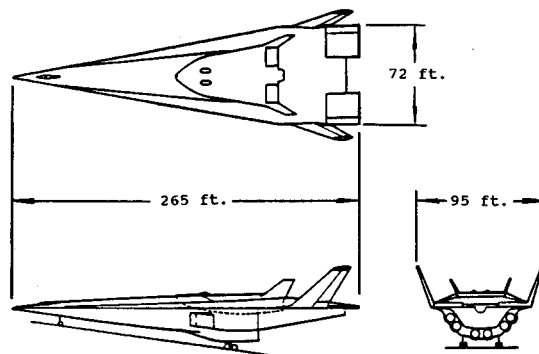


Fig. 1 Oxidizer collecting second-stage launch vehicle.

Received April 26, 1991; revision received Sept. 9, 1991; accepted for publication Sept. 9, 1991; presented as Paper 91-0472 at the AIAA Aircraft Design Systems and Operations Meeting, Baltimore, MD, Sept. 23–25, 1991. Copyright © 1991 by the American Institute of Aeronautics and Astronautics, Inc. All rights reserved.

*Graduate Research Assistant, Department of Mechanical and Materials Engineering. Member AIAA.

†Brage Golding Distinguished Professor, Department of Mechanical and Materials Engineering. Member AIAA.

‡Professor, Department of Mechanical and Materials Engineering. Associate Fellow AIAA.

§Research Engineer, Department of Mechanical and Materials Engineering.

engine performance was expected to suffer as the collected LOX will not be entirely free of N_2 that degrades combustion efficiency. Krieger² examined requirements for single-stage-to-orbit (SSTO) missions and their respective effects on launch vehicle design and performance. The necessity for improved engine performance and reduction in structure weight increases with the severity of the mission requirements, thus driving a need for advances in AB engine development and advanced materials. The use of a horizontal takeoff/landing concept was expected to add a significant amount of weight to an SSTO vehicle over other concepts (expendable launcher, Shuttle-like, etc.), which in turn drives the need for an advance in AB propulsion.

Orton and Mark³ studied potential LOX/hydrocarbon propulsion systems to reduce both the dry weight and fuel tank volume over LOX/ H_2 systems. The hydrocarbon fuels are lower in cost and toxicity and are less corrosive than LOX/ H_2 fuels. In addition, the greater fuel density of the hydrocarbon fuel leads to a smaller fuel tank volume/weight. The propulsion systems were evaluated on the following criteria: \dot{V} capability, $W_{FULLY\ FUELED}$, and W_{EMPTY} . Methane gave the lowest weight for systems designed for a specific \dot{V} or total impulse requirement. LOX/ethanol gave the greatest \dot{V} and total impulse capability due to its greater density-specific impulse product. Ethanol was chosen as the best hydrocarbon fuel based on existing technology for engine development, and it is a clean burning (noncoking) propellant.

Garg et al.⁴ developed an integrated flight/propulsion control system (IF/PCS) to optimize the propulsion system dynamics to augment flight control functions. The control system developed and extended some related work in the field by partitioning the controller into simplified, lower-order subcontrollers that may be utilized independently of each other on the airframe or engine with no significant losses in system performance or robustness. The system leads to a single, high-order integrated feedback compensator that is appealing for "optimum" performance. Smith et al.⁵ developed an adaptive IF/PCS to optimize aircraft flight and inlet control system interaction with engine controllers during steady-state engine operation. The objective of the control system was to minimize fuel consumption during cruise as well as maximizing thrust during aircraft accelerations, climbs, and dashes. Onboard models of the inlet, engine, and nozzle were optimized to produce incremental changes to the nominal engine/inlet control schedules. Simulations indicate realizable and significant increases in thrust (15%) and specific fuel consumption (3%). Schmidt et al.⁶ examined the cross coupling of engine and airframe dynamics and the resultant effect on engine/airframe control system stability and performance. They derived two coupling transfer function matrices that quantified the significance of engine/airframe interaction and showed that when critical coupling terms are small (relative to the magnitude of the loop transfer function), cross coupling effects are minimal.

Murthy and Czysz⁷ addressed the development of high-speed vehicles that are effective in all parts of their trajectories, (i.e., by maximizing energy and energy availability in the vehicle). This is significant in hypersonic aircraft that seek to maximize the use of mass, momentum, and energy in the vehicle propellant, friction-generated and radiative heat in the aircraft and the engine, and the aircraft's exhaust jet and vehicle wake, etc. The paper presented methods of maximizing work output from the vehicle and identified potential improvements in processes, components, and system architecture.

ACES is a viable propulsion concept, and by minimizing the LH_2 fuel volume required to collect the necessary second-stage LOX, the vehicle structure may be reduced with resulting weight savings. With reductions in vehicle structural weight and improved payload capability, a vehicle employing ACES technology is certainly a candidate for development. In this paper, results are presented for a test vehicle to demonstrate the methodology to optimize LOX collection. For this investigation, only the vehicle performance has been optimized. The optimization is performed using a two degree-of-freedom trajectory simulation, suitable for rapid design tradeoff studies. The methodology integrates the equations of motion and tracks velocity, altitude, range, flight-path

angle, and vehicle weight. Significant decreases in fuel burned with optimized LOX collection may be accomplished with the methodology presented next.

Space Booster Vehicle Description

The vehicle chosen for evaluation is a conceptual, two-stage horizontal takeoff/landing vehicle, developed to explore in-flight oxidizer collection. A general view of an oxidizer collecting aircraft, taken from Ref. 1, is shown in Fig. 1. Aerodynamic and propulsion characteristics were provided by the U.S. Air Force and are represented graphically in Figs. 2–5. The objective of this data is to represent values associated with ACES vehicles to develop the performance methodology. No special significance should be given to specific numerical values.

The computed performance was based on the following vehicle properties:

Stage 1 and 2 at takeoff

$$W_{TAKEOFF} = 817,369 \text{ lb}$$

$$W_{FUEL} = 251,000 \text{ lb}$$

$$W_{LOX} = 0.0 \text{ lb}$$

$$S = 11,108 \text{ ft}^2$$

$$A_C = 800 \text{ ft}^2$$

$$FA_{STOIC} = \text{stoichiometric for } H_2/\text{air} = 0.0292$$

$$\alpha_0 = \text{zero lift angle of attack} = 0 \text{ deg}$$

Stage 2 at stage point

$$W_{STAGE2} = 559,631 \text{ lb}$$

$$W_{FUEL} = 27,000 \text{ lb}$$

$$W_{LOX} = 379,000 \text{ lb}$$

$$S = 4098 \text{ ft}^2$$

The system being investigated is subject to constraints determined by the laws of mechanics, aerodynamics, and propulsion. In addition, the vehicle must not exceed specific design constraints imposed by limitations of the structure, materials, flight control, etc. These constraints are entitled inequality constraints and may apply only during portions of the flight. Constraints are imposed on the following parameters:

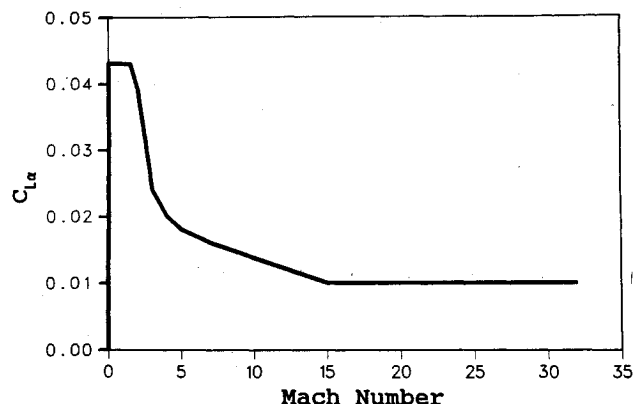


Fig. 2 Lift coefficient vs Mach number.

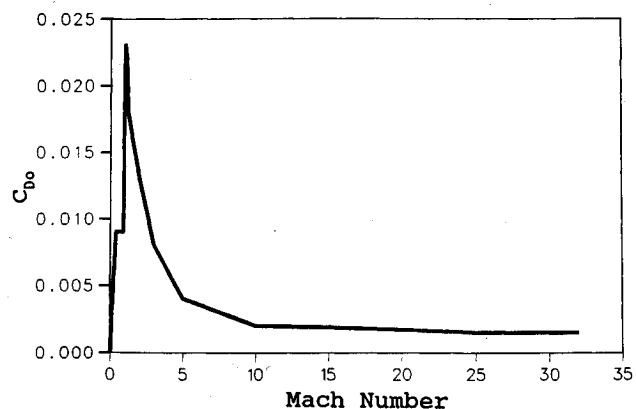


Fig. 3 Zero lift drag coefficient vs Mach number.

Dynamic pressure:

$$q_{\text{LIMIT}} = \frac{1}{2} \rho V^2 = 1000 \text{ lb/ft}^2$$

Acceleration:

$$\dot{V}/g = 1.0$$

$$0 < L/W < 2.0$$

Control effectiveness:

$$\dot{\alpha} \leq 0.05 \text{ rad/s}$$

Propulsion cycle:

$$T = T_{\text{MAX}} \text{ (climb)}$$

$$T = C_{\phi} T_{\text{MAX}} \text{ (cruise, descent)}$$

$$\dot{W}_{\text{MAX}} = 200.0 \text{ lb/s}$$

Air Collection Mission Profile

To accomplish the performance analysis of in-flight oxidizer collection, a typical mission is defined. The primary legs of the mission are identified such that an optimization can be performed. A conceptual oxidizer collection mission and second-stage boost to orbit is divided into nine components that are described as follows:

- 1) Climb to desired energy state along a minimum fuel path.
- 2) Cruise at maximum LOX collection rate/minimum fuel rate until LOX tanks are filled.
- 3) Accelerate to desired staging point (altitude and velocity).
- 4) Vehicle separation.

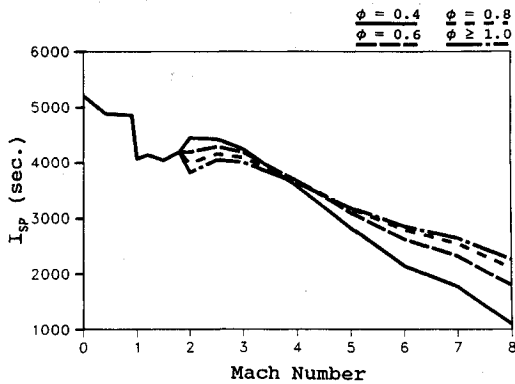


Fig. 4 Hydrogen fueled turbojet/ramjet/scramjet engine, specific impulse (I_{sp} , s).

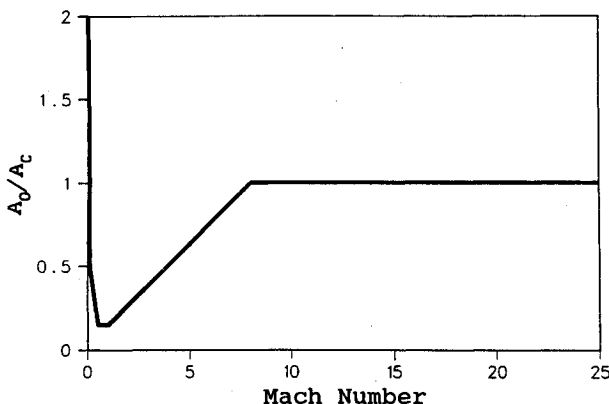


Fig. 5 Engine inlet streamtube/capture area ratio (A_0/A_c).

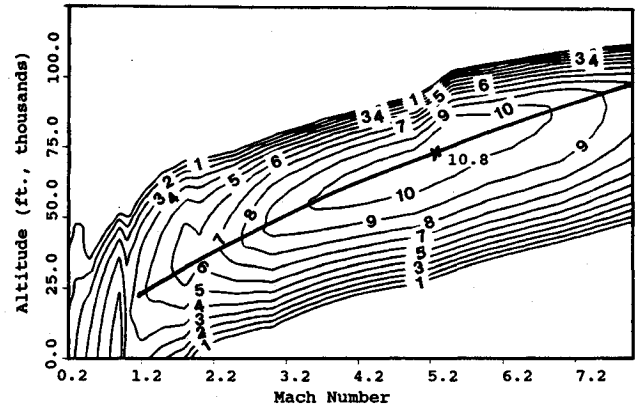


Fig. 6 Fuel specific energy contours (f_s , ft/lb), first stage with LOX collection from $1.5 < M < 5.5$, ACES vehicle ($W = 900,000$ lb).

- 5) First stage return to home station at maximum range condition.
- 6) First stage descend to desired subsonic endurance condition, throttle reduced for minimum fuel descending path.
- 7) First stage loiter at desired subsonic endurance state and land.
- 8) Second stage acceleration to orbital velocity.
- 9) Second stage climb to orbital altitude.

This paper concentrates on minimizing fuel burned during the LOX collection and acceleration to the staging point (i.e., mission components 2 and 3). Optimizing these mission components to minimize fuel burned serves to maximize the potential range factor of the first-stage aircraft, thus providing for a safe fuel allotment for return to the home station, which is an important concern in any successful mission.

By controlling both thrust and angle of attack, the vehicle may be guided along a desired flight path. A robust guidance system (see Kauffman et al.^{8,9}) has been developed to control the stick and throttle settings. The guidance law controls the angle of attack α by a second-order rate control and controls the thrust by limiting the fuel flow \dot{W} and a first-order rate control on the equivalence ratio ϕ . The thrust computation is given as

$$T = I_{sp} F A_{\text{STOIC}} \phi_{\text{MAX}} \dot{W}_{\text{MAX}} C_{\phi} \quad (1)$$

where $0 \leq C_{\phi} \leq 1$.

Optimization

The objective of the optimization is to minimize fuel burned to attain a desired staging condition ($M = 8.0$, fully loaded second-stage LOX tanks) for second-stage ascent to orbit. The relationship for the fuel expended during flight is given as:

$$\Delta W_F = \int \dot{W}_F dt = \int_0^E \frac{\dot{W}_F}{E} dE = \int_0^E \frac{dE}{P_s/\dot{W}} \quad (2)$$

With a definite integral (or isoperimetric) constraint condition imposed on the LOX collection

$$\Delta W_{\text{LOX}} = \int_0^t \dot{W}_{\text{LOX}} dt = \int_0^{\Delta W_F} \frac{\dot{W}_{\text{LOX}}}{\dot{W}_F} dW_F = \text{const} \quad (3)$$

or in terms of LOX collection the equation may be expressed as

$$\Delta W_{\text{LOX}} = \int_0^{\Delta W_F} C_R dW_F = \int_0^E \frac{W}{P_s} C_R dE = \text{const} \quad (4)$$

Equation (2) shows that minimum fuel is accomplished by maximizing the fuel specific energy term ($f_s = P_s/\dot{W}$), which is general

for all types of flight/launch vehicles. Plotting contours of fuel specific energy as a function of altitude and Mach number allows the flight envelop to be determined. The contour map, Fig. 6, immediately identifies the superior areas of performance, where the numerals on the contour map indicate the vehicle's fuel specific energy at that point. For the case of minimum fuel burned, the vehicle should follow the maximum values of f_s between any two energy states; the optimum climb path for minimum fuel for combined first and second stages is shown on the figure.

Once an optimum climb path is established using variational principles (Venugopal et al.¹⁰), it is necessary to determine the optimum region for the collection of the second-stage LOX. The collector hardware imposes a limit of $1.5 \leq M_{\text{COLLECT}} \leq 5.5$ due to limitations on inlet pressure and temperature. The maximum collection ratio ($C_R = \dot{W}_{\text{LOX}}/\dot{W}_{\text{LH}_2}$) that is subject to limitations on refrigeration capability has a maximum value of approximately 6.0. In considering the optimal collection parameters, two strategies may be considered, i.e., collection at a constant Mach number or collection over a variable Mach number range. Either method is controlled by the throttle setting (C_ϕ) that in turn changes the magnitude of the fuel specific energy term f_s .

Collection at Constant Mach Number

In collecting at a constant Mach number, the fuel required may be readily determined from the LOX constraint [Eq. (4)]:

$$\int_0^{\Delta W_F} C_R dW_F = (C_R)_{\text{COLLECT}} (\Delta W_F)_{\text{COLLECT}} \quad (5)$$

Equation 5 may be expressed as fuel burned during LOX collection:

$$(\Delta W_F)_C = \frac{\Delta W_{\text{LOX}}}{(C_R)_{\text{COLLECT}}} \quad (6)$$

The fuel required along other points is determined from the following integration:

$$\Delta W_F = \int_0^t \dot{W}_F dt = \int_{E_1}^{E_2} \frac{dE}{f_s} \quad (7)$$

The fuel requirement for the mission is divided into three parts: 1) fuel required to accelerate from takeoff to start of LOX collection (E_1), 2) fuel expended during LOX collection, and 3) fuel required to accelerate to staging (E_2). The equation for each of the fuel expenditures are given next:

$$\Delta W_{F1} = \int_{E_0}^{E_1} \frac{\dot{W}}{P_S} dE = \frac{E_1 - E_0}{f_s} \quad (8)$$

$$(\Delta W_F)_C = \int \dot{W}_F dt = \int_0^{\Delta W_{\text{LOX}}} \frac{\dot{W}_F}{\dot{W}_{\text{LOX}}} dW_{\text{LOX}} = \frac{\Delta W_{\text{LOX}}}{(C_R)_C} \quad (9)$$

$$\Delta W_{F2} = \int_{E_C}^{E_2} \frac{\dot{W}}{P_S} dE = \frac{E_C - E_2}{f_s} \quad (10)$$

For the given case, total fuel expenditure is minimized where

$$\Delta W_{F_{\text{TOTAL}}} = \Delta W_{F1} + \Delta W_{FC} + \Delta W_{F2} \rightarrow \text{minimum} \quad (11)$$

To permit a simple solution, it was assumed that $f_{S1} = f_{S2} = f(M_C)$. Rewriting Eqs. (8) and (10) for ΔW_F gives the following:

$$\Delta W_F = \frac{(E_2 - E_1)}{f_s} + \frac{\Delta W_{\text{LOX}}}{C_R} = f(M_C) \quad (12)$$

For the collection region of interest between Mach 2.5 and 5.0, the difference $E_2 - E_1 = 296,800$ ft (where $E = h + \frac{1}{2} V^2$, $h_1 = 45,000$ ft, $V_1 = 2420$ ft/s, $h_2 = 68,800$ ft, and $V_2 = 4840$ ft/s). The optimum Mach number for LOX collection is determined by differentiation ($\Delta W_{\text{LOX}} = 379,000$ lb):

$$\frac{d\Delta W_F}{dM} = 0 = \frac{E_2 - E_1}{f_s} \left(\frac{df_s}{dM} \right) - \frac{\Delta W_{\text{LOX}}}{C_R^2} \left(\frac{dC_R}{dM} \right) \quad (13)$$

Solving for the differential ratio of fuel specific energy with respect to Mach number gives

$$\frac{dC_R}{dM} = -0.68 \left(\frac{C_R}{f_s} \right)^2 \frac{df_s}{dM} \quad (14)$$

Using typical values for C_R (5.5) and f_s (Fig. 6, $f_s = 9.0$) and substituting into Eq. (14) produces the following numerical value for the optimum C_R slope:

$$\frac{dC_R}{dM} = -0.5 \quad (15)$$

A plot of the collection ratio vs Mach number is evaluated to determine a collection Mach number for which the slope dC_R/dM brackets the optimum value found in Eq. (15). For the given case, an optimum collection point occurred at $M_{\text{COLLECT}} = 3.5$. The fuel expended for three different collection Mach numbers determined by numerical integration of the trajectory is shown in Table 1, which confirms the simple analysis that $M_{\text{COLLECT}} = 3.5$ is optimum for LOX collection at a constant Mach number.

Collection at Variable Mach Number

The alternative strategy is to collect LOX while accelerating over a range of Mach numbers. As stated previously, the system is capable of collecting LOX for Mach numbers of 1.5–5.5; however, the best collection range lies in the range $2.5 \leq M_{\text{COLLECT}} \leq 5.0$ (Fig. 7). The equation for evaluating fuel burned is given as:

$$\Delta W_{\text{FUEL}} = \int_{E_1}^{E_2} \frac{dE}{f_s} \rightarrow \text{minimum} \quad (16)$$

Table 1 Fuel expenditure during LOX collection: all LOX collection at $M_{\text{COLLECT}} = \text{const}$

M_{COLLECT}	2.5	3.5	5.0
Fuel to collect LOX, lb	80,000	86,977	115,349
Fuel to reach staging point, lb	83,721	73,488	53,953
Total fuel to collect and deliver, lb	163,721	160,465	169,302

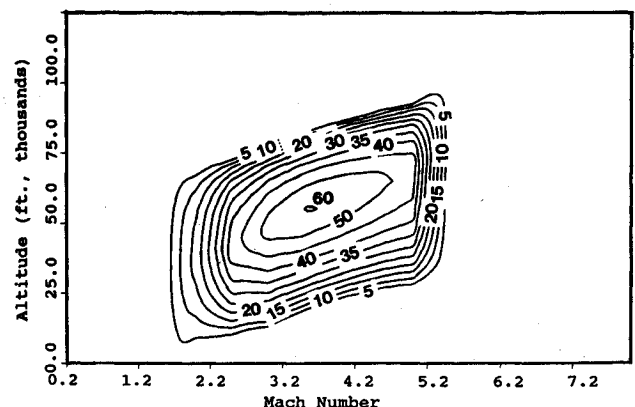


Fig. 7 LOX collection region contours ($C_R \times P_S$, ft-lb), first stage with LOX collection from $1.5 < M < 5.5$, ACES vehicle ($W = 900,000$ lb).

With a constraint of

$$\Delta W_{\text{LOX}} = \int_{E_1}^{E_2} \frac{C_R}{f_s} dE = 379,000 \text{ lb} \quad (17)$$

Variational methods can be used to determine the command value of f_s to satisfy the LOX constraint and minimize the LH₂ fuel expended. To accomplish this, the following substitution is used:

$$\frac{\dot{W}}{P_s} = \left(\frac{\dot{W}}{P_s} \right)_{C_\phi=1} + \sigma(E) \quad (18)$$

The goal is to determine the optimum σ function such that fuel expenditure is minimized. Hence,

$$\Delta W_{\text{LOX}} - \int_{E_1}^{E_2} \left(\frac{\dot{W}}{P_s} \right)_{C_\phi=1} C_R dE \equiv \text{const} \quad (19)$$

$$\Delta W_f - \int_{E_1}^{E_2} \left(\frac{\dot{W}}{P_s} \right) dE = \int_0^{\Delta E} (\sigma) dE \rightarrow \text{minimum} \quad (20)$$

To obtain a solution for σ , Fourier series can be used for approximating σ as a function of E . The following simple expression may be used to describe σ . Let $\sigma = kC_R$, where the constant k is determined by iteration during oxygen collection to attain the desired LOX load. To confirm these findings, a parametric study was conducted to determine the optimum collection rate during accelerated flight. The ϕ schedule to achieve the theoretical optimum collection rate is shown in Fig. 10 based on the derived rate of $\sigma = kC_R$.

Guidance Functions

The LOX collection simulation requires the development of "stick and throttle" controls to closely follow the optimum functions $\sigma(E)$ and $\phi(E)$ that were derived earlier. The pitch attitude is controlled by a second-order rate controller on the angle of attack of the aircraft and is given as follows:

$$\alpha_c = \frac{4}{R_\alpha \tau^3} \left[(h_c - h) + (\dot{h}_c - \dot{h}) \tau + (\ddot{h}_c - \ddot{h}) \frac{\tau^2}{2} \right] \quad (21)$$

In the equation just given, $h_c = f(V)$ is obtained from either the energy method (f_s contours) or from a constant dynamic pressure path. The throttle setting is controlled by a first-order rate controller and is given as follows:

$$\phi_c = \frac{2}{R_\phi \tau_v^2} [(V_c - V) + (\dot{V}_c - \dot{V}) \tau_v] \quad (22)$$

To follow the desired function $\sigma(E)$, the function is converted to an acceleration command as follows:

$$W_{\text{LOX}_{\text{MAX}}} = W_{\text{LOX}_v} + \int_V^{V_F} \frac{\dot{W}_F C_R}{\dot{V}} dV \quad (23)$$

Assuming

$$L_{FCN}(V) = \frac{\dot{W}_F C_R}{\dot{V}} = f(V)_{\text{LINEAR}} \quad (24)$$

$$W_{\text{LOX}_{\text{MAX}}} = W_{\text{LOX}_v} + \frac{L_{FCN}(V_F) + L_{FCN}(V)}{2} (V_F - V) \quad (25)$$

which gives

$$L_{FCN}(V) = \frac{2(W_{\text{LOX}_{\text{MAX}}} - W_{\text{LOX}_v})}{(V_F - V)} - L_{FCN}(V_F) \quad (26)$$

Table 2 Fuel expenditure during LOX collection: begin LOX collection while accelerating to M_{COLLECT} , then remaining LOX is collected at $M_{\text{COLLECT}} = \text{const}$

M_{COLLECT}	3.5	4.5	5.0	5.25
Fuel to collect LOX, lb	74,299	80,734	84,112	126,168
Fuel to reach staging point, lb	75,234	62,724	55,140	49,066
Total fuel to collect and deliver, lb	149,533	143,458	139,252	175,234

Table 3 Fuel expenditure for collect-on-the-run-parametrics

Start collect M_1	End collect M_2	$G_{FCN} V_2$	Fuel expenditure to collect and deliver W_{FUEL} , lb
2.0	5.0	90.0	136,469
2.0	5.0	200.0	136,987
2.5	5.0	50.0	135,055
2.5	5.0	90.0	134,087
2.5	5.0	150.0	134,896
2.5	5.0	200.0	135,817
2.5	4.5	90.0	139,030
2.5	5.25	90.0	133,030
3.0	5.0	90.0	134,430

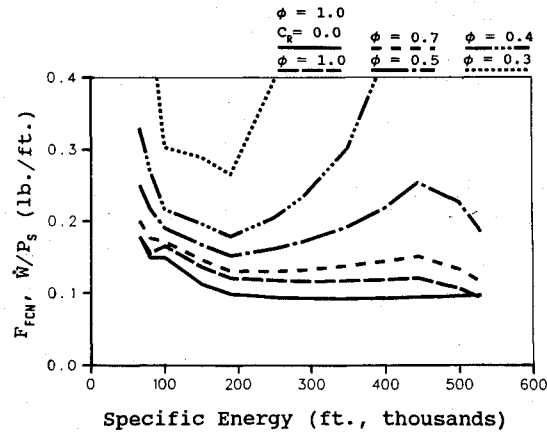


Fig. 8 Fuel integrand (F_{FCN} , lb/ft.).

with

$$\dot{V}_c = \frac{\dot{W}_c}{L_{FCN}(V)} \quad (27)$$

This throttle control was found to be very stable over a large range of operating values and was used to follow the optimal $\sigma(E)$ function previously derived with the following input values:

- V_F = final collection velocity
- $L_{FCN}(V_F)$ = value of (W_{LOX}/V) at V_F
- V_0 = guidance initiation velocity
- τ_v = system gain (4.33 recommended)

The vehicle velocity, acceleration, fuel flow, LOX collected, and R_ϕ are measured or generated during a mission simulation. The throttle control is valid for climb, descent, and cruise conditions and assures that the LOX tanks are full at V_F .

Vehicle Collection Performance

Figures 8 and 9, respectively, show fuel-flow and LOX-flow values divided by specific excess power (P_s), (F_{FCN} and L_{FCN}) for values of fuel/air equivalence ratio ϕ , over the Mach number range $1.5 \leq M_{\text{COLLECT}} \leq 5.5$. The area under any L_{FCN} curve is the total LOX collected, and the area under the corresponding F_{FCN} curve (for a given equivalence ratio value) is the fuel required to collect the LOX load. The values represented in the figures were computed for a constant dynamic pressure path (1000 psf).

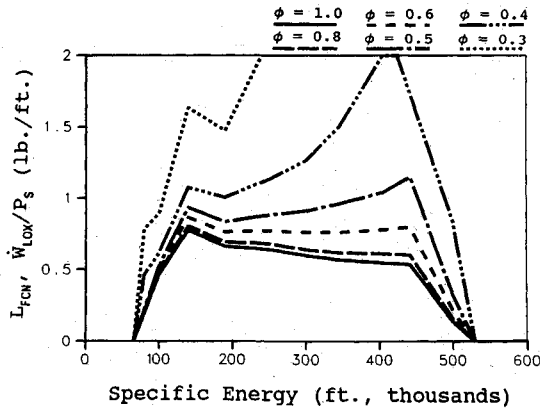
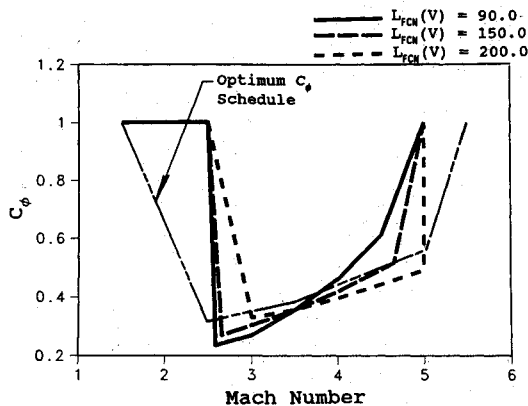
Fig. 9 LOX integrand (L_{FCN} , lb/ft).

Fig. 10 Variable throttle schedule (LOX collection = 379,000 lb).

An example of how to use these curves follows; first the average value of the LOX integrand is determined, which will fill the second-stage tanks for collection between $M_1 = 2.5$ ($E_1 = 142,401$ ft) and $M_2 = 5.0$ ($E_2 = 444,511$ ft), which gives

$$G_{AVG} = \frac{W_{LOX}}{E_2 - E_1} = 1.255 \text{ lb/ft} \quad (28)$$

The corresponding average fuel integrand gives $F_{AVG} = 0.25$ lb/ft; therefore, the fuel used to collect 379,000 lb of LOX is

$$W_F = F_{AVG} (E_2 - E_1) = 75,528 \text{ lb} \quad (29)$$

If this numerical exercise is repeated for several trapezoidal figures, we begin to see that a throttle schedule that starts with a low ϕ (< 0.3) and ends with $\phi = 1.0$ results in minimum fuel used to collect the LOX. However, if the LOX is collected too early, there is a penalty for accelerating the LOX load to $M = 8.0$ (Table 2).

Throttle Parametrics

It is necessary to validate through a parametric study that "optimum" results in minimum fuel required to collect and deliver the necessary LOX load to $M = 8.0$ is achieved. Three different exercises were conducted.

1) The case for LOX collection at constant Mach number is shown in Table 1. The best case occurred at $M = 3.5$ and resulted in a fuel expenditure of 160,000 lb.

2) The second exercise collected LOX while accelerating from $M = 1.5$ to a designated collection Mach number, where the LOX was collected at a constant Mach number. These results are shown

in Table 2, where the best case was found to occur at $M_{COLLECT} = 5.0$ and resulted in a fuel expenditure of 139,000 lb.

3) The third exercise utilized the throttle guidance equations to automatically determine the ϕ schedule that would assure that the LOX tanks were full before $M = 5.5$. The results for this study were obtained by varying three parameters: 1) initiation velocity, 2) final velocity, and 3) the value of the LOX integrand $L_{FCN}(V)$ at the final collection velocity. Figure 10 shows an example of the ϕ schedules generated, and Table 3 shows some tabulated results. The best case fuel expenditure of 133,000 lb was approximately 600 lb greater than that achieved by following an optimum throttle schedule.

Conclusions

A methodology has been developed to conduct rapid trade studies for vehicles using air collection and enrichment systems (ACES) to minimize fuel burned while collecting second-stage LOX. In this paper, a conceptual two-stage vehicle has been analyzed to demonstrate the procedure. The optimum flight path (angle of attack control) was determined to be the locus of fuel specific energy values. The conclusions for this vehicle evaluation are summarized as follows:

- 1) The optimization accomplishes the desired objective, i.e., minimizing fuel to collect and deliver the required LOX load.
- 2) The guidance equations developed follow the calculated optimal results very closely and are easy to implement.
- 3) Collecting LOX while accelerating over the full range of C_R values results in a savings of 27,000 lb of fuel (17%) over LOX collection at constant Mach number.

Acknowledgments

This research was supported by the U.S. Air Force contract F33615-87-C-1550 through the Center for Artificial Intelligence Applications, Dayton, Ohio. The technical support of the Aeropropulsion and Power Directorate at Wright Laboratory is greatly appreciated.

References

1. Nau, R. A., "A Comparison of Fixed Wing Reusable Booster Concepts," Society of Automotive Engineers, SAE Paper 670384, Palo Alto, CA, May 1967.
2. Krieger, R. J., "A Summary of Features and Design Issues for Single-Stage-to-Orbit Vehicles," AIAA Paper 90-1932, July 1990.
3. Orton, G. F., and Mark, T. D., "LOX/Hydrocarbon Auxiliary Propulsion for the Space Shuttle Orbiter," *Journal of Spacecraft and Rockets*, Vol. 21, No. 6, 1984, pp. 580-586.
4. Garg, S., Mattern, D. L., and Bullard, R. E., "Integrated Flight/Propulsion Control System Design Based on a Centralized Approach," *Journal of Guidance, Control, and Dynamics*, Vol. 14, No. 1, 1991, pp. 107-116.
5. Smith, R. H., Chisholm, J. D., and Stewart, J. F., "Optimizing Aircraft Performance with Adaptive, Integrated Flight/Propulsion Control," ASME Paper 90-GT-252, Brussels, Belgium, June 1990.
6. Schmidt, D., Schierman, J., and Garg, S., "Analysis of Airframe/Engine Interactions—An Integrated Control Perspective," AIAA Paper 90-1918, July 1990.
7. Murthy, S. N. B., and Czysty, P., "An Approach to Air-Breathing High Speed Vehicle Synthesis," AIAA Paper 91-0225, Jan. 1991.
8. Kauffman, H. G., Grandhi, R. V., Hankey, W. L., and Belcher, P. J., "Improved Airbreathing Launch Vehicle Performance with the Use of Rocket Propulsion," *Journal of Spacecraft and Rockets*, Vol. 28, No. 2, 1991, pp. 172-178.
9. Kauffman, H. G., Grandhi, R. V., Hankey, W. L., and Belcher, P. J., "System Optimization for Maximizing Reconnaissance Mission Range of a Hypersonic Cruise Vehicle," AIAA Paper 90-3292, Sept. 1990.
10. Venugopal, N., Grandhi, R. V., Hankey, W. L., and Belcher, P. J., "Combined Energy Management And Calculus of Variations Approach For Optimizing Hypersonic Vehicle Trajectories," *Journal of Computing Systems in Engineering*, Vol. 1, No. 2-4, 1990, pp. 591-600.

James A. Martin
Associate Editor